

Worst-case performance analysis for SDF-based dataflow MoCs

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Outline I

Introduction

Dataflow

Synchronous dataflow (SDF)

SDF-based parametrized dataflow (SDF-PDF)

Performance indicators for dataflow graphs

Max-plus semantics of self-timed execution of SDF

Max-plus semantics of self-timed execution of SDF-PDF

Worst-case performance for SDF-PDF

Case study

Questions

References

Models of Computation (MoC)[10]

- Models of Computation (MoCs) describe system behaviour by defining
 - System components and an execution model for components and
 - System communication model, i.e. how the information is exchanged between components

Models of Computation (MoC)

- Given a system, the challenge is to find a good MoC for that system
- This is the key to efficient design and analysis

Models of Computation (MoC)

- Time plays a role
- The Von Neumann MoC?
- Thread based concurrency models?

Models of Computation (MoC)

- For control-dominated and reactive systems
 - State-based models are appropriate, monitor inputs, change state and set outputs
- For data-dominated systems
 - Actor-based dataflow models are appropriate, transform input data streams into output data streams

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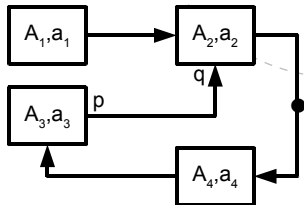
Case study

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References

Dataflow MoCs

- Instantiated as directed graphs
- Nodes are called *actors*
 - Describe some functionality
- Edges are called *channels*
 - Describe actor dependencies
- Token
 - A quantum of information exchanged



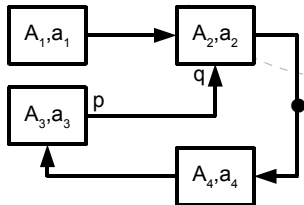
```

fire {
    ...
    token = InPort.Read();
    ...
    OutPort.Write(token);
    ...
}

```


Dataflow MoCs

- Actor firing
 - Quantum of computation
 - Token consumption and production
- Rates
 - Token production and consumption numbers
 - Actors as processes, token sequences as signals
 - Describe signal partitioning
- Actor firing delay
 - Actor execution time



```

fire {
    ...
    token = InPort.Read();
    ...
    OutPort.Write(token);
    ...
}

```

Dataflow Advantages

- Simple graphical representation
- Elegantly expresses parallelism
- An intuitive system representation for signal processing and multimedia designers
- Best software practices: modularity and code reuse
- Software synthesis from dataflow graphs (front end)

Dataflow MoC categories

- Static dataflow MoCs
- Dynamic dataflow MoCs
- The former can describe dynamic application, the latter cannot

Outline I

Introduction

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Max-plus semantics of self-timed execution of SDF-PDF

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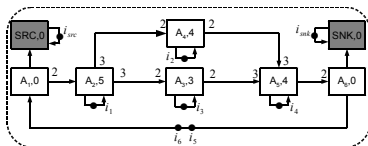
Case study

Questions

References

Synchronous dataflow (SDF)

- Introduced by Edward Lee et al., UC Berkley, 1987 [9]
- Most widely used dataflow MoC in general
- Rates and actor firing delays are known at compile time
- Channels as FIFOs



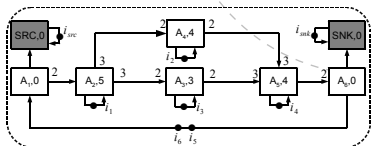
Synchronous dataflow (SDF)

— Initial tokens

- Typically understood to be an initial condition for an execution, rather than part of the execution

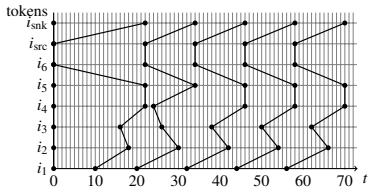
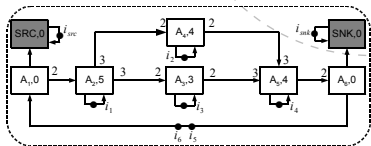
— Progresses in iterations

- A sequence of actors' firings \mathcal{I} that brings the SDF graph (SDFG) to its initial state
- Typically a coherent set of calculations, e.g. decoding a video frame
- Graph repetition vector Γ



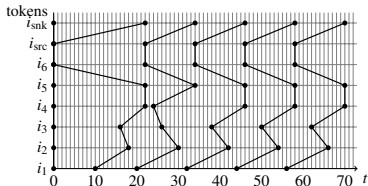
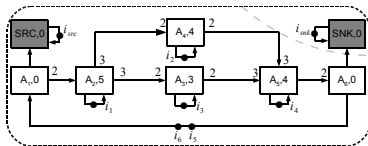
Synchronous dataflow (SDF)

- Graph repetition vector:
 $\Gamma(SRC, A_2, A_3, A_4, A_5, A_6, SNK) = (1, 1, 2, 3, 3, 2, 1, 1)$
- Graph iteration:
 $\mathcal{I} = A_1^1 SRC_1^1 A_2^2 A_3^3 A_4^3 A_5^2 A_6^1 SNK_2^1$
- Self-timed execution
- Ultimately periodic



Synchronous dataflow (SDF)

- Advantages of SDF
 - Compile-time predictable/analyzable
 - High implementation efficiency
- Disadvantages of SDF
 - Low expressive power



Outline I

Introduction

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Max-plus semantics of self-timed execution of SDF-PDF

Worst-case performance for SDF-PDF

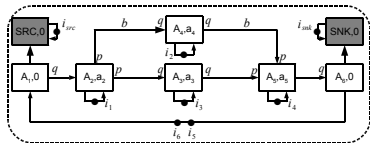
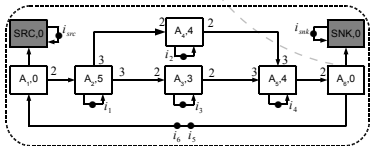
Case study

Questions

References

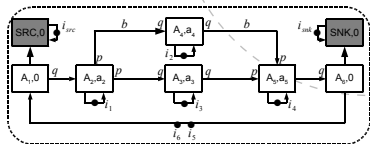
Parametrized dataflow

- Integrate dynamic parameters and run-time adaptation of parameters in a structured way into a certain dataflow MoC called the *base model*
- Base models with well-defined concept of a graph iteration, e.g. SDF
- SDF-based parametrized dataflow (SDF-PDF)
 - SDF parametrization

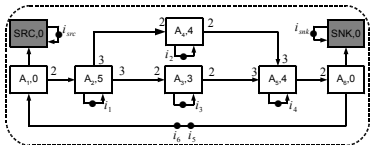


SDF-PDF

- Parameter value become known at run-time, i.e. SDF-PDFG is configured at run-time
- SDF-PDFG configuration c is obtained by assigning values to all parameters of the SDF-PDFG
- When we apply c to the SDF-PDFG, an SDFG emerges
- This SDFG is called the instance of the original SDF-PDFG, denoted $\iota_G(c)$



$$\downarrow c = \{p = 3, q = 2, b = \tau\tau, a_2 = 5, a_3 = 3, a_4 = 4, a_5 = 4\}$$



SDF-PDF

- SDF-PDFG domain

$$X = \{c_k\}, k \in \mathbb{N}_{>0}$$

- Graph repetition vector:

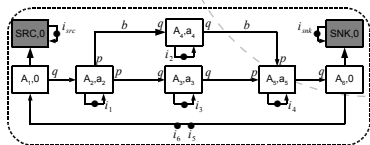
$$\Gamma(\text{SRC}, A_2, A_3, A_4, A_5, A_6, \text{SNK}) = (1, 1, q, p, p, q, 1, 1)$$

- Graph iteration:

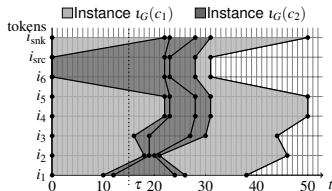
$$\mathcal{I} = A_1^1 \text{SRC}_1^1 A_2^p A_3^q A_4^q A_5^p A_6^1 \text{SNK}_2^1$$

- An SDF-PDF (SDF-PDFG) progresses in base model iterations

- Parameter values change between iterations



$$\downarrow c_1 = \{p=3, q=2, b=tt, a_2=5, a_3=3, a_4=4, a_5=4\}, c_2 = \{p=1, q=1, b=ff, a_2=2, a_3=3, a_4=1, a_5=1\}$$



SDF-PDF

- SDF-PDF is a collective name for parametrized dataflow MoCs where SDF serves as the base models and not another MoC
- Parametrized SDF (PSDF) [3], schedulable parametric dataflow (SPDF) [4], boolean parametric dataflow (BPDF) [2] and variable rate dataflow (VRDF) [11] are example of such models

Outline I

Introduction

Dataflow

Synchronous dataflow (SDF)

SDF-based parametrized dataflow (SDF-PDF)

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Worst-case performance for SDF-PDF

Case study

Questions

References

Throughput

- The amount of work a system can perform over a period of time
- We consider dataflow models with well-defined concept of a graph iteration
- Throughput Th is the long run-average number of completed iterations per time-unit

Worst-case throughput of an SDF-PDFG

- One iteration of an SDF-PDFG corresponds to one iteration of an arbitrary SDFG instance

Worst-case throughput (SDF-PDFG)

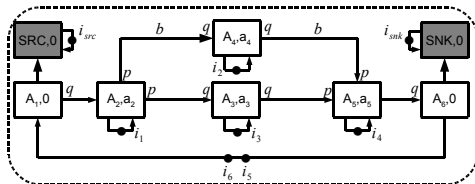
The worst-case throughput of an SDF-PDFG G is given as

$$Th(G) = \min_{\sigma \in \Sigma} \liminf_{k \rightarrow \infty} \frac{\sigma_k}{C_k^\sigma}$$

where σ is a sequence of SDFG instances, Σ is the set of all admissible sequences of SDFG instances, σ_k is the first k elements of σ and C_k^σ is the completion time of σ_k .

Latency

- The time delay between the stimulus and its effects
- In dataflow, stimuli and their effects are actor firings
- One is typically interested in the maximal time-span between firings of actors that represent the input and output of the system
- These actors typically have a repetition vector entry of one
- If not, we consider groups of firing of those actors that contain as many firings as their repetition vector entries
- We record these groups using the *SRC* and *SNK* actors



Worst-case latency of an SDF-PDFG

Worst-case latency (SDF-PDFG)

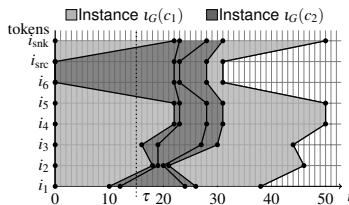
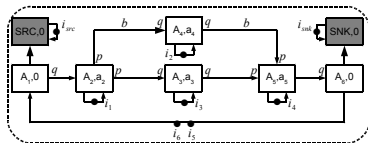
The worst-case latency of an SDF-PDFG G is given as

$$L(G) = \max_{\sigma \in \Sigma} \max_{k \in \mathbb{N}_{>0}} (F_{SNK,k}^{\sigma} - F_{SRC,k}^{\sigma})$$

where σ is a sequence of SDFG instances, Σ is the set of all admissible sequences of SDFG instances and $F_{A,k}^{\sigma}$ is the completion time of the k -th firing of an SDF-PDFG actor A .

Challenges

- SDF-PDFG actors execute in parallel within a graph iteration;
- SDF-PDFG iterations overlap, i.e. they are pipelined;
- SDF-PDFG iterations are inter-dependent, i.e. synchronized by the availability of the initial tokens;
- SDF-PDFG is a dynamic dataflow structure, i.e. properties of consecutive iterations may drastically differ.



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Introduction

Dataflow

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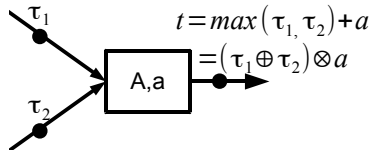
Case study

Questions

References

Synchronization and delay

- Self-timed execution of SDFGs
 - Synchronization
 - Delay
- Synchronization corresponds to the \max operator, while delay corresponds to the \oplus operator
 \Rightarrow Max-plus algebra



Max-plus algebra

- A semiring defined over $\mathbb{R}_{\max} = \mathbb{R} \cup \{-\infty\}$
- $a \oplus b = \max(a, b)$ and $a \otimes b = a + b$
- $-\infty \oplus a = a \oplus -\infty = a$ and $a \otimes -\infty = -\infty \otimes a = -\infty$
- $A \oplus B$ is defined by $[A \oplus B]_{ij} = a_{ij} \oplus b_{ij}$
- $A \otimes B$ is defined by $[A \otimes B]_{ij} = \bigoplus_{j=1}^k a_{ij} \otimes b_{jk}$
- We use $a \otimes c$ or $c \otimes a$ to denote the vector $[a_i + c]$
- \otimes symbol in the exponent indicates a matrix power, $c \in \mathbb{R}$,
 $c^{\otimes n} = n \cdot c$

Max-plus for SDF

- SDFGs progress in iterations
- Define a vector $\gamma(k)$ that records the production times of initial tokens after the k -th SDFG iteration
- Then, the evolution of the graph can be described using the following Max-plus linear equation

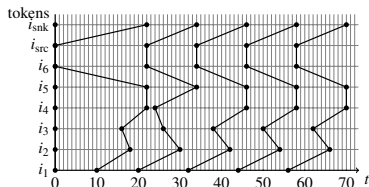
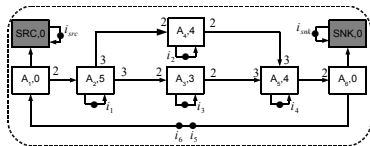
$$\gamma(k+1) = M_G \otimes \gamma(k)$$

- M_G is the Max-plus characteristic matrix of an SDFG and is obtained by symbolic simulation of the graph for a single iteration [6]

Max-plus for SDF

$$\gamma(k) = [t_{i_1}, t_{i_2}, t_{i_3}, t_{i_4}, t_{i_5}, t_{i_6}, t_{i_{src}}, t_{i_{snk}}]^T, \quad \gamma(0) = [0, 0, 0, 0, 0, 0, 0, 0]^T$$

$$\gamma(1) = \underbrace{\begin{bmatrix} 10 & -\infty & -\infty & -\infty & -\infty & 10 & -\infty & -\infty \\ 18 & 12 & -\infty & -\infty & -\infty & 18 & -\infty & -\infty \\ 16 & -\infty & 9 & -\infty & -\infty & 16 & -\infty & -\infty \\ 22 & 16 & 14 & 8 & -\infty & 22 & -\infty & -\infty \\ 22 & 16 & 14 & 8 & -\infty & 22 & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & 0 & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & 0 & 0 & -\infty \\ 22 & 16 & 14 & 8 & -\infty & 22 & -\infty & 0 \end{bmatrix}}_{M_G} \otimes \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \max(10, 10) \\ \max(18, 12, 18) \\ \max(16, 9, 16) \\ \max(22, 16, 14, 8, 22) \\ \max(22, 16, 14, 8, 22) \\ \max(0) \\ \max(0, 0) \\ \max(22, 16, 14, 18, 22) \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ 16 \\ 22 \\ 22 \\ 0 \\ 0 \\ 22 \end{bmatrix}$$

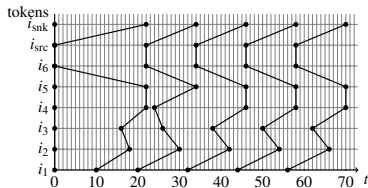


Max-plus for SDF

- Sequence $\bar{\gamma} = \gamma(1), \gamma(2), \dots$ is ultimately periodic
- From the cyclicity theorem of Max-plus algebra it follows that for every $k \geq t(M_G)$

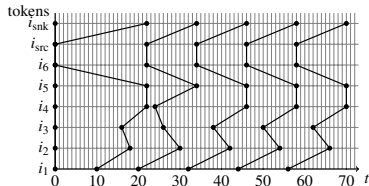
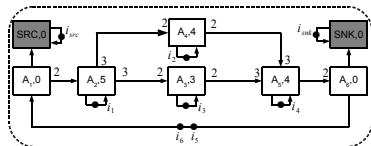
$$\gamma(k + \nu) = \lambda^{\otimes \nu} \otimes \gamma(k)$$

where ν is the cyclicity of M_G , λ the eigenvalue of M_G and $t(M_G)$ the transient time of M_G



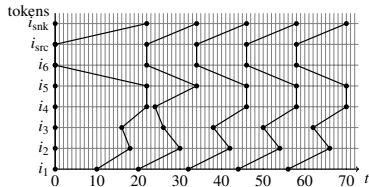
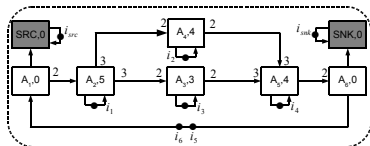
Throughput of an SDFG

- For an SDFG G , $Th(G) = 1/\lambda$
- The value of λ corresponds to the maximum cycle mean (MCM) of the communication graph of M_G
- $Th(G) = 1/12$ iterations per time-unit



Latency of an SDFG

- Let S_γ be the set of the first $t(M_G) + v$ elements of $\bar{\gamma} = \gamma(1), \gamma(2), \dots$
- For an SDFG G , $L(G) = \max_{\gamma \in S_\gamma} ([\gamma]_{i_{\text{snk}}} - [\gamma]_{i_{\text{src}}})$
- $L(G) = 22 \text{ time-units}$



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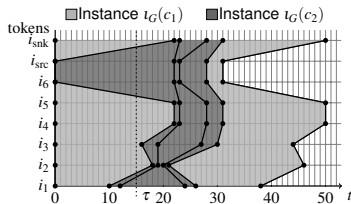
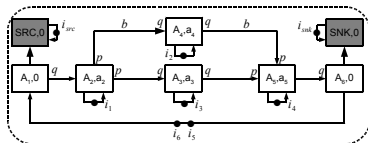
Case study

Questions

References

Max-plus for SDF-PDF

- An SDF-PDFG progresses in SDFG instance iterations, i.e. one SDF-PDFG iteration corresponds to an iteration of an arbitrary SDFG instance $\iota_G(c)$ of that SDF-PDFG
- Thus, $\gamma(k+1) = M_{\iota_G(c)} \otimes \gamma(k)$
- Can we actually compute all $\{M_{\iota_G(c)}\}$ where $c \in X$?



Max-plus for SDF-PDF

- We would like to obtain a parametrized Max-plus characteristic matrix of an SDF-PDFG $M(c)$ where $c \in X$
- $M(c)$ for a concrete c becomes $M_{IG(c)}$
- Symbolic simulation of the graph for a single iteration?
- Analytical simulation of the graph for a single iteration?
- The goal is to express the dependence of $\gamma(k+1)$ on $\gamma(k)$

$$X \xrightarrow{p \quad l \quad q} Y$$

$$\tau(Y, n) = \tau \left(X, \left\lceil \frac{n \cdot q - l}{p} \right\rceil \right) \otimes y$$

Max-plus for SDF-PDF

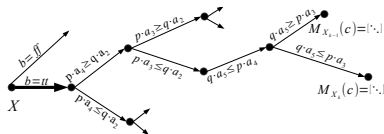
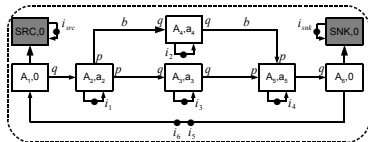
- $\gamma(k+1) = [t'_{i_1}, t'_{i_2}, t'_{i_3}, t'_{i_4}, t'_{i_5}, t'_{i_6}, t'_{i_{\text{src}}}, t'_{i_{\text{snk}}}]^T$
- $\gamma(k) = [t_{i_1}, t_{i_2}, t_{i_3}, t_{i_4}, t_{i_5}, t_{i_6}, t_{i_{\text{src}}}, t_{i_{\text{snk}}}]^T$
- t_{i_1} corresponds to the vector $[0, -\infty, -\infty, -\infty, -\infty, -\infty, -\infty, -\infty]^T$
all the way to $t_{i_{\text{snk}}}$ that corresponds to the vector $[-\infty, -\infty, -\infty, -\infty, -\infty, -\infty, -\infty, 0]^T$
- The iteration schedule: $\mathcal{J} = A_1^1 \text{SRC}_1^1 A_2^p A_3^q A_4^q A_5^p A_6^1 \text{SNK}_2^1$

$$X \xrightarrow{p \quad l \quad q} Y$$

$$\tau(Y, n) = \tau \left(X, \left\lceil \frac{n \cdot q - l}{p} \right\rceil \right) \otimes y$$

Max-plus for SDF-PDF

- Max-plus superposition
- Max-plus convolution
- Conditional channels



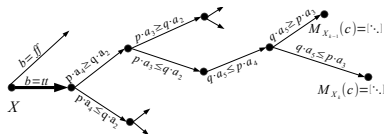
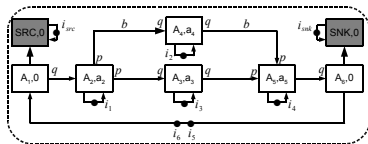
Max-plus for SDF-PDF

— Fire A_1

$$\begin{aligned}\tau(A_1, 1) &= t_{i_6} \otimes 0 \\ &= [-\infty, -\infty, -\infty, -\infty, -\infty, 0, -\infty, -\infty] \otimes \gamma(k)\end{aligned}$$

— Fire A_2

$$\begin{aligned}\tau(A_2, n) &= t_{i_6} \otimes a_2^{\otimes n} \oplus t_{i_1} \otimes a_2^{\otimes n} \\ &= [a_2^{\otimes n}, -\infty, -\infty, -\infty, -\infty, a_2^{\otimes n}, -\infty, -\infty] \otimes \gamma(k)\end{aligned}$$

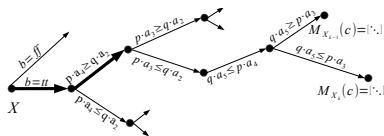
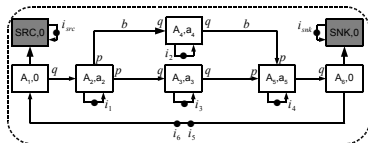


Max-plus for SDF-PDF

— Fire A_4

$$\tau(A_4, n) = [a_2^{\otimes (\frac{q}{p} + 1)} \otimes a_4^{\otimes n},$$

$$a_4^{\otimes n}, -\infty, -\infty, -\infty, a_2^{\otimes (\frac{q}{p} + 1)} \otimes a_4^{\otimes n}, -\infty, -\infty] \otimes \gamma(k) \quad (1)$$



Max-plus for SDF-PDF

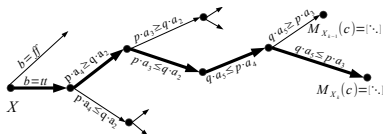
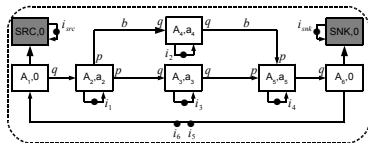
- After completing the iteration, we collect the results into the vector $\gamma(k+1) = [t'_{i_1}, t'_{i_2}, t'_{i_3}, t'_{i_4}, t'_{i_5}, t'_{i_6}, t'_{i_{src}}, t'_{i_{snk}}]^T$ where

$$t'_{i_1} = \tau(A_2, q), \quad t'_{i_2} = \tau(A_4, p)$$

$$t'_{i_3} = \tau(A_3, p), \quad t'_{i_4} = \tau(A_5, q)$$

$$t'_{i_5} = \tau(A_6, 1), \quad t'_{i_6} = t_{i_5}$$

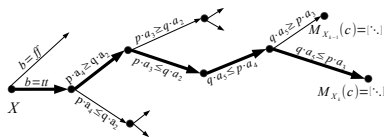
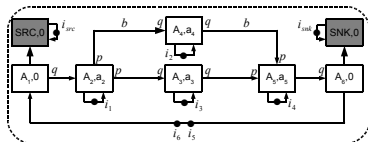
$$t'_{i_{src}} = \tau(SRC, 1), \quad t'_{i_{snk}} = \tau(SNK, 1)$$



Max-plus for SDF-PDF

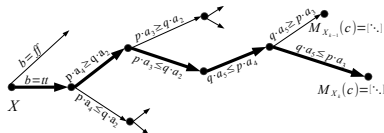
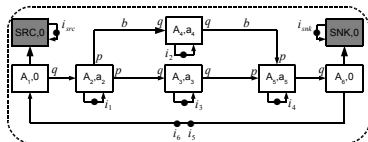
— We obtain

$$M_{X_k}(c) = \begin{bmatrix} a_2^{\otimes q} & -\infty & -\infty & -\infty & -\infty & a_2^{\otimes q} & -\infty & -\infty \\ a_2^{\otimes(\frac{q}{p}+1)} \otimes a_4^{\otimes p} & a_4^{\otimes p} & -\infty & -\infty & -\infty & a_2^{\otimes(\frac{q}{p}+1)} \otimes a_4^{\otimes p} & -\infty & -\infty \\ a_2^{\otimes(q+1)} \otimes a_3 & -\infty & a_3^{\otimes p} & -\infty & -\infty & a_2^{\otimes(q+1)} \otimes a_3 & -\infty & -\infty \\ a_2^{\otimes(\frac{q}{p}+1)} \otimes a_4^{\otimes(p+1)} \otimes a_5 & a_4^{\otimes(p+1)} \otimes a_5 & a_3^{\otimes(p+1)} \otimes a_5 & a_5^{\otimes p} & -\infty & a_2^{\otimes(\frac{q}{p}+1)} \otimes a_4^{\otimes(p+1)} \otimes a_5 & -\infty & -\infty \\ a_2^{\otimes(\frac{q}{p}+1)} \otimes a_4^{\otimes(p+1)} \otimes a_5 & a_4^{\otimes(p+1)} \otimes a_5 & a_3^{\otimes(p+1)} \otimes a_5 & a_5^{\otimes p} & -\infty & a_2^{\otimes(\frac{q}{p}+1)} \otimes a_4^{\otimes(p+1)} \otimes a_5 & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & 0 & -\infty & -\infty & -\infty \\ -\infty & -\infty & -\infty & -\infty & -\infty & 0 & 0 & -\infty \\ a_2^{\otimes(\frac{q}{p}+1)} \otimes a_4^{\otimes(p+1)} \otimes a_5 & a_4^{\otimes(p+1)} \otimes a_5 & a_3^{\otimes(p+1)} \otimes a_5 & a_5^{\otimes p} & -\infty & a_2^{\otimes(\frac{q}{p}+1)} \otimes a_4^{\otimes(p+1)} \otimes a_5 & -\infty & 0 \end{bmatrix}$$



Max-plus for SDF-PDF

- Initial X has to be partitioned into a set of subregions $\{X_k\}$ that we call natural subdomains of an SDF-PDFG
- Complexity issues
- Conservativeness of $M_{X_k}(c)$
 - $x \leq \lceil x \rceil < x + 1$ with respect to $\tau(Y, n) = \tau\left(X, \lceil \frac{n \cdot q - l}{p} \rceil\right) \otimes y$ and monotonicity of SDF



Outline I

Introduction

Dataflow

Synchronous dataflow (SDF)

SDF-based parametrized dataflow (SDF-PDF)

Performance indicators for dataflow graphs

Max-plus semantics of self-timed execution of SDF

Max-plus semantics of self-timed execution of SDF-PDF

Worst-case performance for SDF-PDF

Case study

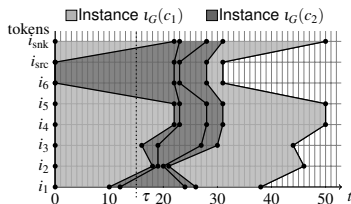
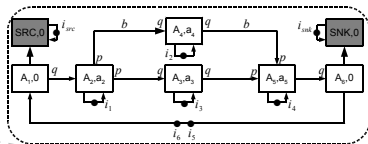
Questions

References

Worst-case evaluation of an SDF-PDFG

- For and SDF-PDFG, $\gamma(k+1) = M_{\iota_G(c)} \otimes \gamma(k)$, i.e.
 $\gamma(k+1) = M_{X_k}(c) \otimes \gamma(k)$ s.t. $c \in X_k$
- SDF-PDFG, FSM-SADF [7] and Max-plus automata [5]
- Instances occur in an arbitrary order
- Worst-case evaluation Max-plus matrix:

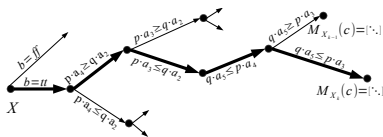
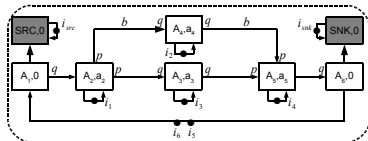
$$M = \bigoplus_{c_k \in X} M_{\iota_G(c_k)}$$



Worst-case evaluation of an SDF-PDFG

- X is too large to be enumerated
- Worst-case evaluation Max-plus matrix re-written:

$$M = \bigoplus_{X_k \in X} \bigoplus_{c \in X_k} M_{X_k}(c)$$



Worst-case evaluation of an SDF-PDFG

- Worst-case evaluation Max-plus matrix re-written:

$$M = \bigoplus_{X_k \in X} \bigoplus_{c \in X_k} M_{X_k}(c)$$

- Obtaining $\bigoplus_{c \in X_k} M_{X_k}(c)$:

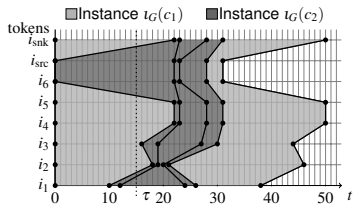
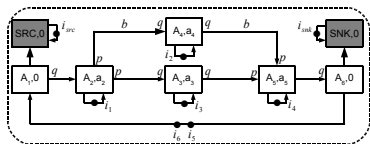
foreach (i,j) s.t. $[M_{X_k}(c)]_{ij} \neq -\infty$ **do**

maximize $[M_{X_k}(c)]_{ij}$
 p

subject to $c \in X_k$.

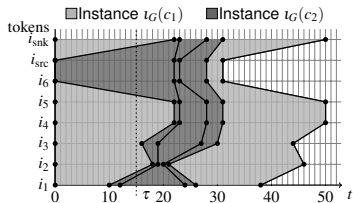
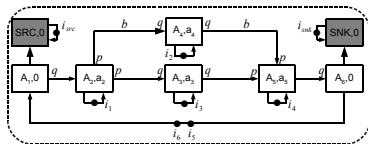
Worst-case throughput of an SDF-PDFG

- For an SDF-PDFG G , $Th(G) = 1/\lambda$
- The value of λ corresponds to the maximum cycle mean (MCM) of the communication graph of M



Worst-case latency of an SDF-PDFG

- Let S_γ be the set of the first $t(M) + v$ elements of $\bar{\gamma} = \gamma(1), \gamma(2), \dots$
- For an SDF-PDFG G , $L(G) = \max_{\gamma \in S_\gamma} ([\gamma]_{i_{\text{snk}}} - [\gamma]_{i_{\text{src}}})$



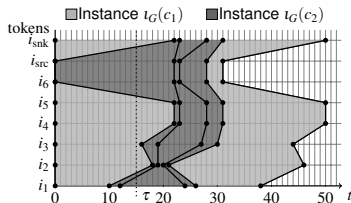
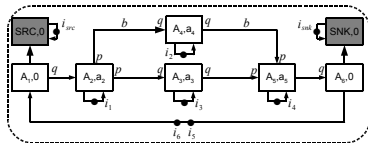
Discussion

- If all parameters are constrained to intervals, then isn't it simpler to create a "worst-case SDFG" from an SDF-PDFG by taking the interval upper endpoints for values of the parameters?
- The answer is yes, because SDF is monotonic
- However, what if parameters expose interdependencies?
- Rates and firing delays
- Then, taking the upper endpoints might result in serious over-approximations
- These reduce design's optimization potential

Discussion

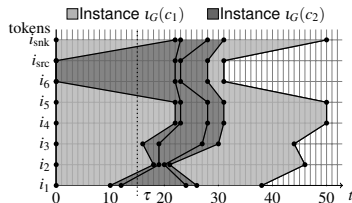
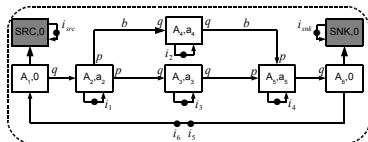
— Assume:

$$X = X_k \cap \{p = w_1 \cdot w_2, w_1 + w_2 = 2 \cdot x_1 - x_2, \\ p \in [1, 10], q \in [1, 10], w_1 \in [1, 3], w_2 \in [1, 4], \\ x_1 \in [1, 3], x_2 \in [1, 5], a_2 \in [1, 7], a_4 = 4, \\ a_3 \in [1, 5], a_5 = 4\}$$



Discussion

Comparison			
Metric	SDF	SDF-PDF	$\Delta_{\text{rel}} [\%]$
Throughput [<i>it. per time-unit</i>]	1/70	1/24	291.66
Latency [<i>time-units</i>]	79.0	42.5	46.20



Outline I

Introduction

Dataflow

Synchronous dataflow (SDF)

SDF-based parametrized dataflow (SDF-PDF)

Performance indicators for dataflow graphs

Max-plus semantics of self-timed execution of SDF

Max-plus semantics of self-timed execution of SDF-PDF

Worst-case performance for SDF-PDF

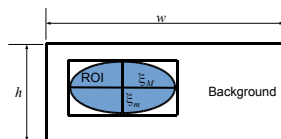
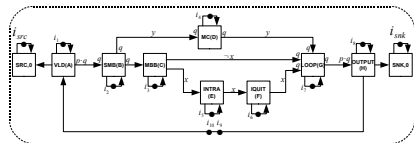
Case study

Questions

References

VC-1 decoder

- VC-1 decoder [1]
- ROI coding [8]



$$X = \{p = (2 \cdot \xi_M \cdot 2 \cdot \xi_m) / (16 \cdot 16), p \in [1, P], q \in [1, 16]\}$$

$$p' \geq \mu / 100 \cdot P, p + p' \leq P$$

$$o^2 = 4 \cdot \pi^2 (\xi_M^2 + \xi_m^2), o \geq O, \varepsilon^2 \cdot \xi_M^2 = \xi_M^2 - \xi_m^2, \varepsilon = E, 2 \cdot \xi_M \leq w, 2 \cdot \xi_m \leq h$$

$$x \wedge \neg y \mid \neg x \wedge y \mid x \wedge y,$$

$$a = 7400, b = 10, c = 10, d = 1937, e = 288, f = 365, g = 4074, h = 10\}$$

Outline I

Introduction

Dataflow

Synchronous dataflow (SDF)

SDF-based parametrized dataflow (SDF-PDF)

Performance indicators for dataflow graphs

Max-plus semantics of self-timed execution of SDF

Max-plus semantics of self-timed execution of SDF-PDF

Worst-case performance for SDF-PDF

Case study

Questions

References

Questions

Thanks! Questions?

Outline I

Introduction

Dataflow

Synchronous dataflow (SDF)

SDF-based parametrized dataflow (SDF-PDF)

Performance indicators for dataflow graphs

Max-plus semantics of self-timed execution of SDF

Max-plus semantics of self-timed execution of SDF-PDF

Worst-case performance for SDF-PDF

Case study

Questions

References

References I



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