Intelligence and Reference

Formal ontology of the natural computation

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Summary

1. **Turing seminal work:**
   - From the Algorithmic Computation (AC) paradigm
   - To the Natural Computation (NC) paradigm

2. **The paradigmatic case of reference:**
   - From Formal Logic (AC case: representationalism)
   - To Formal Ontology (NC case: realism)

3. **The dual ontology underlying NC:**
   - From the infinitistic scheme: math → phys law → information
   - To the finitistic scheme: information → math → phys law

4. **The Mutual Re-definition between Numbers and Processes (MRNP) and its applications in NC:**
   - Geometric Perceptron (AC) vs. Dynamic Perceptron (NC)
   - Extrinsic non-computable (AC) vs. Intrinsic computable (NC) chaotic dynamics characterization
   - Applications to cognitive neurosciences

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TURING SEMINAL WORK:
FROM AC TO NC PARADIGM

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Section I
After his fundamental work on AC paradigm (1936), Turing worked for widening the notion of «computation»:

- **1939**: «Oracle Machine(s) (OTM)», TM enriched with the outputs of non-TM computable functions like as many TM basic symbols, and their transfinite hierarchy;
- **1942**: anticipation of connectionist ANN, i.e., computational architectures made by undefined interacting elements, suitable for statistical training;
- **1952**: mathematical theory of «morphogenesis»: model of pattern formation via non-linear equations – in the case, chemical reaction-diffusion equations simulated by a computer.
NC Paradigm vs. AC Paradigm

- 5 main dichotomies (Dodig-Crnkovic 2012a,b):
  1. **Open, interactive agent-based computational systems (NC) vs. closed, stand-alone computational systems (AC);**
  2. **Computation as information processing and simulative modeling (NC) vs. computation as formal (mechanical) symbol manipulation (AC);**
3. **Adequacy of the computational response via self-organization** as the main issue (NC) in computability theory vs. halting problem (and its many, equivalent problems) as the main issue (AC);

4. Intentional, object-directed, pre-symbolic computation, based on chaotic dynamics in neural computation (NC) vs. representational, solipsistic, symbolic computation, based on linear dynamics typical of early AI approach to cognitive neuroscience (AC).
5. **Dual ontology** based on the energy-information distinction in natural (physical, biological and neural) systems (NC) vs. monistic ontology based on the energy-information equivalence in all natural systems (AC)
Towards New Foundations in Computability Theory

† Necessity of New Foundations in Computability Theory for making complementary these dichotomies (like wave theory and corpuscular theory of light in quantum mechanics), by considering in one only relation structure both causal and logical relations, as the same notion of Natural (i.e., «causal» process) Computation (i.e., «logical» process) suggests (see also the cognitive neuroscience slogan: «from synapses to rules»).

† A typical case of such a required complementarity is the reference problem:
  – in logic, between meta-language and object-language;
  – in epistemology and ontology, between logical and extra-logical (physical, conceptual) entities.
Section II

THE CASE OF REFERENCE: FROM FORMAL LOGIC (AC) TO FORMAL ONTOLOGY (NC)

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Reference in formal semantics and AC (OTM)

- Tarski 1935:
  - Not only the meaning but also the reference in logic has nothing to do with the real, physical world. To use the classic Tarski’s example, the semantic reference of the true atomic statement “the snow is white” is not the whiteness of the crystalized water, but at last an empirical set of data to which the statement is referring, eventually taken as a primitive in a given formal language (≈ OTM in AC and Ramsey’s “ramified type theory” in Logic).
Methodological solipsism and representationalism

- Logic is always representational, it concerns relations among tokens, either at the symbolic or sub-symbolic level. It has always and only to do with representations, not with real things.
- R. Carnap’s (1936) principle of the methodological solipsism in formal semantics extended by H. Putnam (1975) and J. Fodor (1980) to the representationalism of the functionalist cognitive science based on symbolic AI, according to the AC paradigm.
- W.V.O. Quine’s (1960) opacity of reference beyond the network of equivalent statements meaning the same referential object in different languages.
After Searle’s *Chinese Room* another room metaphor Putnam suggested for emphasizing AC limitations in semantics: to solve the simplest problem of how many objects are in «this» room: three (a lamp, a chair, a table) or many trillions (if we consider the molecules) and ever much more (if we consider also atoms and sub-atomic particles…)

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Out of metaphor, any computational procedure of a TM (and any AC procedure at all, if we accept the Turing-Church thesis) supposes the determination of the basic symbols on which the computations have to be carried on – the partial domain on which the recursive computation has to be carried on.

Hence, from the semantic standpoint, any computational procedure supposes that such numbers are encoding (i.e., unambiguously naming as rigid designators) as many “real objects” of the computation domain.

In short, owing to the coding problem, the determination of the basic symbols (numbers) on which the computation is to be carried on, cannot have any computational solution in the AC paradigm.
Putnam theory of causal reference...

- Putnam’s abandon of representationalism in cognitive science for a particular approach to the intentionality theory closer to the Aristotelian one than to the phenomenological one, in which intentionality is related with the «causal» continuous redefinition of basic symbols for the best matching with the outer reality (Latin intellectus as «thinking»), on which further computations/deduction as rule-following symbolic processing are based (Latin ratio (reasoning) as «thought»).

- Putnam indeed rightly vindicated that a causal theory of reference supposes that at least at the beginning of the social chain of “tradition” of a given denotation there must be an effective causal relation from the denoted thing to (the cognitive agent producing) the denoting name/number – and, in the limit, in this causal sense must be intended also the act of perception Kripke vindicated as sufficient for the dubbing of a given object.
What is necessary is a “causal”, “finitistic” theory of coding in which the real thing causally and progressively determines the partial domain of the descriptive function recursively denoting it.

- Necessity of a **formal ontology as a particular interpretation of modal logic relational structures**, for formalizing such an approach to the meaning/reference problem in the NC paradigm.

- I.e., Necessity of a **formal calculus of relations** able to include in the same, coherent, formal framework both “causal” and “logical” relations, as well as the “pragmatic” (real, causal relations of real world with and among the cognition/computation/communication agents), and not only the “syntactic” (logical relations among terms) “semantic” (logical relations among symbols) components of meaningful actions/computations/cognitions.

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Following (Blackburn, de Rijke & Venema, 2010) we can distinguish three eras of modal logic (ML) recent history:

1. **Syntactic era (1918-1959):** C.I.Lewis…
2. **Classic era (1959-1972):** S. Kripke’s… relational semantics based on frame theory
3. **Actual era (1972…):** S. K. Thomason’s algebraic interpretation of modal logic \(\rightarrow\) ML as fundamental tool in theoretical computer science
   a. **Correspondence principle:** equivalence between modal formulas interpreted on models and first order formulas in one free variable \(\rightarrow\) Possibility of using ML (decidable) for **individuating novel decidable fragments** of first-order logic (being first-order theories (models) incomplete or not fully decidable)
   b. **Duality theory** between ML relation semantics and algebraic semantics based on the fact that models in ML are given not by substituting free variables with constants like in predicate calculus, but by **using binary evaluation letters** in relational structures (frames) like in algebraic semantics.
Modal logic in theoretical computer science and NC paradigm

- Despite such a continuity (Standard Translation(ST)) between ML and Classical (mathematical and predicate) Logic (CL), the peculiarity of ML as to CL, overall for foundational aims in the context of NC paradigm, is well defined in the following quotation, making the relationship between ML and CL similar to that between quantum and classical mechanics (with similar «correspondence» and «duality (complementarity)» principles working in both realms).

- This is related with the foundational interpretation of «computation» using the relational notion of program as a Labeled Transition System (LTS), which interprets computations as «passing through» the state transitions constituting the LTS, and it is the basis for the so called «computational metaphor» in fundamental physics – emphasizing once more that the core foundational problem in computability theory is the «labeling» problem, i.e., the problem of a suitable «counter» of partial recursive functions easily interpretable, on its turn, in the framework of relational structures/semantics.
ML and NC paradigm

- «ML talks about relational structures in a special way ‘from the inside’ and ‘locally’. Rather than standing outside a relational structure and scanning the information it contains from some celestial vantage point, modal formulas are evaluated inside structures, at a particular state. The function of the modal operators is to permit the information stored at other states to be scanned – but crucially only the states accessible from the current point via an appropriate transition may be accessed in this way… (We can) picture a modal formula as a little automaton standing at some state in the relational structure, and only permitted to explore the structure by making journeys to neighboring states» (Blackburn, de Rijke and Venema 2010, xii)
Because of ST, we can use the more intuitive, original approach to ML, intended as the common syntax of all intensional logics, granted that the results we obtain «from the inside» via ML can be translated into CL predicative formulas of AC, even though not the constitution process leading to such results.

ML relational structures with all its intensional interpretations are what is today defined as philosophical logic (Burgess 2009), as far as it is distinguished from the mathematical logic, the logic based on the extensional calculus, and the extensional notions of meaning, truth, and identity.

What generally characterizes intensional logic(s) as to the extensional one(s) is that neither the extensionality axiom nor the existential generalization axiom

\[ A \leftrightarrow B \Rightarrow A = B \quad P \alpha \Rightarrow \exists x P x \]

of the extensional predicate calculus hold in intensional logic(s). Consequently, also the Fegean notion of extensional truth based on the truth tables does not hold in the intensional predicate and propositional calculus.
Intensional logic and intentionality

- There exists an intensional logical calculus, just like there exists an extensional one, and this explains why both mathematical and philosophical logic are today often quoted together within the realm of computer science.

- This means that intensional semantics and even the intentional tasks can be simulated artificially («third person» simulation of «first person» tasks, like in human simulation of understanding, without conceptual «grasping»).

- The “thought experiment” of Searle’s “Chinese Room” is becoming a reality, as it happens often in the history of science.

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Main intensional logics

- **Alethic logics**: they are the descriptive logics of “being/not being” in which the modal operators have the basic meaning of “necessity/possibility” in two main senses:
  - Logical necessity: the necessity of lawfulness, like in deductive reasoning
Ontic necessity: the necessity of causality, that, on its turn, can be of two types:

- Physical causality: for statements which are true (i.e., which are referring to beings existing) only in some possible worlds.
- Metaphysical causality: for statements which are true of all beings in all possible worlds, because they refer to properties or features of all beings such beings.
More…

- **The deontic logics**: concerned with what “should be or not should be”, where the modal operators have the basic meaning of “obligation/permission” in two main senses: *moral* and *legal obligations*.

- **The epistemic logic**: concerned with what is “science or opinion”, where the modal operators have the basic meaning of “certainty/uncertainty”.

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Main axioms of ML syntax

- For our aims, it is sufficient here to recall that formal modal calculus is an extension of classical propositional, predicate and hence relation calculus with the inclusion of some further axioms:
  - \( N: \langle (X \to \alpha) \Rightarrow (\Omega X \to \Omega \alpha) \rangle \), where \( X \) is a set of formulas (language), \( \Omega \) is the necessity operator, and \( \alpha \) is a meta-variable of the propositional calculus, standing for whichever propositional variable \( p \) of the object-language. \( N \) is the fundamental *necessitation rule* supposed in any normal modal calculus.
More…

D: \(<\Box \rightarrow \Diamond >\), where \(\Diamond\) is the possibility operator defined as \(\neg \Omega \neg a\). D is typical, for instance, of the deontic logics, where nobody can be obliged to what is impossible to do.

T: \(<\Box \rightarrow >\). This is typical, for instance, of all the alethic logics, to express either the logic necessity (determination by law) or the ontic necessity (determination by cause).

4: \(<\Box \rightarrow \Box \Diamond >\). This is typical, for instance, of all the “unification theories” in science where any “emergent law” supposes, as necessary condition, an even more fundamental law.

5: \(<\Diamond \rightarrow \Box \Diamond >\). This is typical, for instance, of the logic of metaphysics, where it is the “nature” of the object that determines necessarily what it can or cannot do.
Main Modal Systems

- By combining in a consistent way several modal axioms, it is possible to obtain several **modal systems** which constitute as many **syntactical structures available for different intensional interpretations**.

- So, given that **K** is the fundamental modal systems, constituted by the ordinary propositional calculus **k** plus the necessitation axiom **N**, some interesting modal systems for our aims are: **KT4 (S4, in early Lewis’ notation)**, typical of the physical ontology; **KT45 (S5, in early Lewis’ notation)**, typical of the metaphysical ontology; **KD45 (Secondary S5)**, with application in deontic logic, but also in epistemic logic, in ontology, and hence in NC, as we see.
Generally, in the *alethic* (either logical or ontological) interpretations of modal structures the necessity operator $\Omega p$ is interpreted as “$p$ is true in all possible world”, while the possibility operator $\Diamond p$ is interpreted as “$p$ is true in some possible world”. In any case, the so called reflexivity principle for the necessity operator holds in terms of axiom $T$, i.e., $\Omega p \rightarrow p$.

This is not true in *deontic* contexts. In fact, “if it is obligatory that all the Italians pay taxes, does not follow that all Italians really pay taxes”, i.e., $Op \nRightarrow p$.
In fact, the obligation operator $O \! p$ must be interpreted as “$p$ is true in all ideal worlds” different from the actual one, otherwise $O = \Omega$, i.e., we should be in the realm of metaphysical determinism where freedom is an illusion, and ethics too. The reflexivity principle in deontic contexts, able to make obligations really effective in the actual world, must be thus interpreted in terms of an optimality operator $O_p$ for intentional agents $x$, i.e.,

$$(O \! p \rightarrow \! p) \iff ((O_p (x, p) \land c_a \land c_{ni}) \rightarrow \! p)$$
Reflexivity in epistemic context

In similar terms, in **epistemic** contexts, where we are in the realm of representations of the real world. The interpretations of the two modal epistemic operators \( B(x, p) \), “\( x \) believes that \( p \)”, and \( S(x, p) \), “\( x \) knows that \( p \)” are the following:

- \( B(x, p) \) is true iff \( p \) is true in the realm of representations believed by \( x \).
- \( S(x, p) \) is true iff \( p \) is true for all the **founded** representations believed by \( x \).

Hence the relation between the two operators is the following:

\[
S(x, p) \Leftrightarrow (B(x, p) \land F)
\]
Finitistic and not finistic interpretations

- So, for instance, in the context of a *logicist* ontology, such a $F$ is interpreted as a supposed actually infinite capability of human mind of attaining the logical truth. We will offer, on the contrary, a different *finitistic* interpretation of $F$ within NC.
Reflexivity in epistemic logic

- While
  \[ B(x, p) \rightarrow p \]
  \[ S(x, p) \rightarrow p \]

- because of F

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Kripke relational semantics

- Kripke relational semantics is an evolution of Tarski formal semantics, with two specific characters: 1) it is related to an intuitionistic logic (i.e., it considers as non-equivalent excluded middle and contradiction principle, so to admit coherent theories violating the first one), and hence 2) it is compatible with the necessarily incomplete character of the formalized theories (i.e., with Gödel theorems outcome), and with the evolutionary character of natural laws not only in biology but also in cosmology.

- In other terms, while in Tarski classical formal semantics, the truth of formulas is concerned with the state of affairs of one only actual world, in Kripke relational semantics the truth of formulas depends on states of affairs of worlds different from the actual one (= possible worlds).

- Stipulatory character of Kripke’s possible worlds
Kripke notion of frames

- Kripke notion of frame main novelty in logic of the last 50 years \( \rightarrow \) relational structure.
- This is an ordered pair, \(<W, R>\), constituted by a domain \( W \) of possible worlds \( \{u, v, w\ldots\} \), and a by a two-place relation \( R \) defined on \( W \), i.e., by a set of ordered pairs of elements of \( W \) \( (R \subseteq W \times W) \), where \( W \times W \) is the Cartesian product of \( W \) per \( W \).
- E.g. with \( W = \{u, v, w\} \) and \( R = \{uRv\} \), we have:
Relations defined on frames

Seriality: \(<(\text{om } u)(\text{ex } v)(uRv)>\)
Euclidean property

\[<(\text{om } u)(\text{om } v)(\text{om } w)(uRv \text{ et } uRw \Rightarrow vRw)>\]
Of course, this procedure of a (logical) equivalence constitution by iteration of a transitive and serial (=causal) relation can be extended indefinitely:
KD45 as a secondary S5 (KT45)

S5(KT45)

KD45
In any referential expression we suppose the *extensional* identification between a variable and a constant, like when we identify in a substitutional way a proper name with its definite description (i.e., from «Plato is a teacher» to «Plato is *the* teacher of Aristotle»), in the first case ‘is’ is for ‘∈’ in the second one for ‘=‘):

\[(\exists xFx \land (x = a)) \rightarrow Fa\]
Tarski theorem and reference

- In other term $Fa$ in any referential expression must be intended as a **descriptive function** (like ‘$\sin x$’ in math) that is rightly symbolized in logic as $R'x$.
- In fact, as Tarski theorem emphasizes, $Rxy$ is the relation $R$ between a generic teacher $x$ and a generic pupil $y$, $R'ab$ denotes the unique mastership between $a$ and $b$.
- Hence, if $R$ is a two place function $R(x,y)$, $R'$ must be at least a three place function because it must have the same function $R$ as its proper argument, i.e.: $R'(R,a,b)$, and hence it must be defined in an higher order language $L'$ as to $Rab$. Of course, for demonstrating the referential power of $R'$ (as well as the truth of the meta-language in $L'$) we need $R''$ (and a meta-meta-language in $L'''$), and so indefinitely (see second Goedel theorem)
S/P identity in designations as double saturation betw non-well defined set

- Possible escape way (see Fefermann observation of a consistent interpretation of second Goedel theorem only by including intensional notions):
  - Rigid designation as identity between an argument and its descriptive function: \( a = Ra \) (= fixed point in a dynamic logic procedure).
  - Typical case of using ML (in our case KD45) for individuating decidable fragments in first order predicate logic (effective only for unary predicate domains via their local check)

\[
(\exists x Fx \land (x \not\in a) \rightarrow Fa)
\]
Dynamic reading of the procedure: rigid designation as a dynamic locking
Causal theory of rigid designation: an ancestor

- Science, indeed, depends on what is object of science, but the opposite is not true: hence the relation through which science refers to what is known is a causal [real not logical] relation, but the relation through which what is known refers to science is only logical [rational not causal]. Namely, what is knowable (scibile) can be said as “related”, according to the Philosopher, not because it is referring, but because something else is referring to it. And that holds in all the other things relating each other like the measure and the measured, … (Aquinas, Q. de Ver., 21, 1. Square parentheses and italics are mine).
In another passage, this time from his commentary to Aristotle book of *Second Analytics*, Aquinas explains the singular reference in terms of a “one-to-one universal”, as opposed to “one-to-many universals” of generic predications.

It is to be known that here “universal” is not intended as something predicated of many subjects, but according to some adaptation or adequation (adaptationem vel adaequation) of the predicate to the subject, as to which neither the predicate can be said without the subject, nor the subject without the predicate (In Post.Anal., I,xi,91. Italics mine).
Dual ontology...

- Information and energy as two non superposable physical magnitudes, one immaterial, the other material:
  - It from bit. Otherwise put, every 'it' — every particle, every field of force, even the space-time continuum itself — derives its function, its meaning, its very existence entirely — even if in some contexts indirectly — from the apparatus-elicited answers to yes-or-no questions, binary choices, bits. 'It from bit' symbolizes the idea that every item of the physical world has at bottom — a very deep bottom, in most instances — an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes–no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and that this is a participatory universe (Wheeler, 1990, p. 75)
Both Davies and myself we follow it, together with the great majority of physicists, and generally this position is traced back to Rolf Landauer, who affirmed that “the universe computes in the universe” and not in some Platonic heaven, according to the ontology of the logic realism.

– A point of view, Davies continues, motivated by his insistence that “information is physical”. (...) In other words, in a universe limited in resources and time – for example, in a universe subject to the cosmic information bound - concepts such as real numbers, infinitely precise parameter values, differentiable functions and the unitary evolution of the wave function (as in Zeh or in Tegmark approach, we can add) are a fiction: a useful fiction to be sure, but a fiction nevertheless (Davies, 2010, p. 82)..
Now, according to Davies, the main theoretical consequence of such an ontic interpretation of information that can be connoted as a true change of paradigm in modern science, is the turnaround of the “platonic” relationship, characterizing the Galilean-Newtonian beginning of the modern science:

- **Mathematics → Physical Laws → Information**
- into the other one, Aristotelian, much more powerful for its heuristic power:
- **Information → Mathematics → Physical Laws**
Mutual determination between process and numbers

- Davies is here referring in particular to a series of publications of the physicist Paul Benioff —especially (Benioff, 2002; 2005); but see also more recent (Benioff, 2007; 2012).
- He, by working during the last ten years on the foundations of computational physics applied to quantum theory, envisaged a method of mutual determination between numbers and physical processes. A. L. Perrone ad myself already defined a similar method during the 90’s of last century in a series of publications on the foundations of mathematics, and we applied it mainly to the complex and chaotic systems characterization (Perrone, 1995; Basti & Perrone, 1995; 1996).
Benioff’s position...

- In this way, Benioff can express the core of its method, by generalizing it to whichever abstract physic-mathematical theory, as far as it can be characterized as a structure defined on the complex number field $C$:
  - The method consists in replacing $C$ by $C_n$, which is a set of finite string complex rational numbers of length $n$ in some basis (e.g., binary) and then taking the limit $n\to\infty$. In this way, one starts with physical theories based on numbers that are much closer to experimental outcomes and computational finite numbers than are $C$ based theories (Benioff, 2005, p. 1829).

- In fact, Benioff continues,
  - the reality status of system properties depends on a downward descending network of theories, computations, and experiments. The descent terminates at the level of the direct, elementary observations. These require no theory or experiment as they are uninterpreted and directly perceived. The indirectness of the reality status of systems and their properties is measured crudely by the depth of descent between the property statement of interest and the direct elementary, uninterpreted observations of an observer. This can be described very crudely as the number of layers of theory and experiment between the statement of interest and elementary observations. The dependence on size arises because the descent depth, or number of intervening layers, is larger for very small and very large systems than it is for moderate sized systems (Benioff, 2005, p. 1834).
… and what is lacking

- Of course, what is lacking in such a synthesis of Benioff method is that the length of the finite decimal expansion of the rational numbers concerned, at each layer of the hierarchy, is a variable length as a function of the uncertainty “gap” to be fulfilled, on its turn newly finite.

- Only by a theory of multi-layered dynamic re-scaling, the space $R_n$, defined on rational numbers with a finite, but variable decimal expansion, can approximate, for the infinite limit, the space $R$ of the real numbers of abstract mathematics.
So, by using the new symbol “∅” for denoting the concrete dynamic identity between generic and singular individuals, instead of the abstract static identity denoted by the usual “=”, we can consistently substitute “=” in any occurrence both of definite description formulas in semantics, and in any occurrence of the existence predicate in ontology, because of the actually finite and virtually infinite character of the procedure. E. g., in formal ontology, we have:

\[(\forall^k A)(\forall y A) \square_c (E!(a) \rightarrow (a \in A y) \land ((a \lor y) \in A))\]
Section IV

THE MUTUAL RE-DEFINITION BETWEEN NUMBERS AND PROCESSES (MRNP) AND ITS APPLICATIONS IN NC

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Limitations of linear ANN

Rosenblatt geometric perceptron scheme

Impossibility of parallel calculus in this architecture (Minsky & Papert (1988))
Application: hadronic event
What characterizes a chaotic dynamics is its complex behavior. I.e.,

1. Its unpredictability on a deterministic basis:
   Such systems are able, on a deterministic and hence reproducible basis (e.g., generated by a set of differential equations) to jump on the same unstable orbit, after an unpredictably long transient in which the dynamics visits other unstable orbits.
2. Its instability. A chaotic attractor can be characterized as a folding of unstable orbits of any length.

\[ |x(t_0) - x(t_0 + \tau_i)| \leq \varepsilon \]

\[ \tau_i = \tau_0, \tau_1, \tau_2, \ldots \]

I.e., these unstable cycles can be also of a very high order, so that the time sequences of a chaotic signal could be confused with random ones.
Chaos as folding of unstable cycles

\[
\begin{align*}
\dot{x} &= f(x, y, z) \\
\dot{y} &= g(x, y, z) \\
\dot{z} &= h(x, y, z)
\end{align*}
\]
Dynamic and dissipative chaos
The same idea of DP on time

- Let $X_i (i = 1, \ldots, N)$ be the trajectory generated from the chaotic system from which we want to extract or to stabilize or to synchronize a pseudo-cyclic point of a generic period $\rho$.
- From the given trajectory, we extract periodic cycles which pass near a fixed target $X_t$.
- In order to reduce the number of the sampled (observed) points needed for extraction, we apply the dynamic re-definition of the observation interval.
Computationally...  

- **Computationally** we use the difference of distances from each point $X_i$ to the target $X_t$. The difference of distances at the time step $i$, $D_i$, is defined as follows:

$$D_i = |X_i - X_t| - |X_{i-1} - X_t|$$

- If $D_i < 0$ ($D_i > 0$) then the orbit is **approaching** to (leaving from) the target ($X_t$) at step $i$. We observe the trajectory at the consecutive steps $T_n$ ($n = 1, 2, ...$). These observation steps $T_n$ are defined by the following equation:
where \( \tau_n \) is an observation window relative to the \( n \)-th observation; this window is re-defined for each observation step according to the following equation:

\[
\tau_n = \begin{cases} 
F[k_n] & \text{if } D_{T_{n-2}} < 0 \text{ and } D_{T_{n-1}} < 0 \\
 k_n = k_{n-1} & \text{if } k_{n-1} > 0 \ (k_n = 0 \text{ otherwise}) \\
 k_n = k_{n-1} + 1 & \text{otherwise}
\end{cases}
\]

\[
T_n = T_{n-1} + \tau_n
\]
More…

\[ F[k] = F[k-1] + F[k-2] \]
\[ F[1] = 1 \]
\[ F[0] = 1 \]

– where \( T[0] = 0 \) and \( k[0] = 0 \). When we observe that \( D_{T_n} > 0 \) and \( D_{T_{n-1}} < 0 \), then we search for the step such that \( D_i < 0 \) and \( D_{i+1} > 0 \).
Results on Lorenz attractor
One cycle reconstructed with less points than the original
Chaotic NN as model of neural plasticity

- **A: Instability**
  - Same stimulus $\rightarrow$ several interpretations

- **B: Non-stationarity**
  - Several interpretations $\rightarrow$
    - same final state $\equiv$
    - semantic (content related)
    - definition of a new class

- **A+B: reversibility**
  - $\rightarrow$ Output = pseudo-cycle
  - $\rightarrow$ Possibility of implementing logical calculi in chaotic neural nets

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Dynamic basis of intentionality

- **Chaos as composite TM:**
  - Non-determinist TM = TM quintuples with non-superposable codomains (same input → many outputs)
    
    ```
    x \rightarrow y
    x \rightarrow y
    x \rightarrow y
    ```
  - Irreversibile TM = quintuples with non-superposable domains many inputs → same output)
    
    ```
    x \rightarrow y
    x \rightarrow y
    x \rightarrow y
    ```
Dynamical Basis of intentionality

- Globally a composite MT will produce reversible behaviors (= logical calculi) but **impredictable** because it will follow always different trajectories for different contexts → semantic NN.
- Dissipative function of goals (reducing the possibility space, dissipation of free energy)

\[ \text{X} \quad \text{X} \quad \text{X} \quad \text{X} \quad \text{Y} \]
Informational Richness of Chaos

- So the **informational richness** of chaos.
  - Is naturally associated with the quasi-periodic cycle structure of a complex chaotic dynamics.
  - The following figure exemplifies intuitively the amazing possibilities of memory storing and of dynamic integration of information that a chaotic dynamics in principle owns.
An Hybrid Implementation of a Chaotic Net
Representational vs. Intentional

- **CS development**: from representational and extensional to intentional and intensional.
- **Representational approach**: knowledge as representation (in set theory sense), i.e., functional correspondence environment-brain (→ human mind is **passive**: symbols pre-constituted by evolution and culture: truth as *aequatio*, functional identity satisfaction: \( y = f(x) \) → **functionalism**

*Shanghai 2011*
Intentional vs. Representational

Intentional approach: knowledge as self-modification (*actio immanens*) of the dispositional states to action of the organism toward the environment in order to pursue a goal.

- **Truth as ad-aequatio**, modification of dynamic/inductive categories intended as **dispositions to action (virtual forms or habits)** by which assimilating ourselves to reality for the “maximum grip” to it.
- → Human mind is **active**. Only in a secondary way “calculates” on symbols already constituted (*secondary reflection, reasoning, representational thought*), but primarily it is continuously (re-)constituting them on the outer reality to satisfy human “rational instinct to truth” (*first reflection, intellect, intentional thinking*).
W. Freeman’s mesoscopic approach to neural basis of intentionality

- Intentional approach requires real time (≈10 msec) integration of neuron activity very far among them.
- Basal activity of CNS is not noise to be filtered, it is **stochastic chaos** integrating in real time far neuron activations - i.e., oscillators with different thresholds resonating selectively with one of the multiple frequencies present in a chaotic activation wave.
- Recognition as self-organization (formation/destruction) in real time of non-local lower dimension attractors (similar to condensation/evaporation reaction).
- “Higher part of motor neurons do not code single movements, but motor acts, i.e., movements coordinated by goal pursuing” (Rizzolatti & Sinigaglia, 2006)
Chaotic NN as model of neural plasticity

- **A: Instability**
  - Same stimulus $\rightarrow$ several interpretations

- **B: Non-stationarity**
  - Several interpretations $\rightarrow$ same final state $\equiv$
    $\equiv$ semantic (content related) definition of a new class

- **A+B: reversibility**
  - $\rightarrow$ Output $=$ pseudo-cycle
  - $\rightarrow$ Possibility of implementing logical calculi in chaotic neural nets

Birmingham 2012
Cerbral Implementation

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Intentional Dynamics of Neural Fields
(chaotic neural wave functions at mesoscopic level)

Problem: how is it possible this real-time interaction among neurons very far among them?

- Possibility of modulation
  - In frequency (FM)
  - In amplitude (AM)

- Chaotic neural wave functions for propagating activations simultaneously on many frequencies among far neurons as oscillators with different and changing thresholds

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Microscopic/mesoscopic transition

Layer | Connection type | Level
--- | --- | ---
receptors | pulse frequency | microscopic
olfactory bulb | pulse and wave densities | Phase transition
olfactory cortex | | mesoscopic

updated memory readout
Formation of chaotic attractors in olfactory bulb dynamics

Contour plots of rms amplitudes to show AM patterns and their changes with conditioning.
Conclusion

1. **Turing seminal work:**
   - From the Algorithmic Computation (AC) paradigm
   - To the Natural Computation (NC) paradigm

2. **The paradigmatic case of reference:**
   - From Formal Logic (AC case: representationalism)
   - To Formal Ontology (NC case: realism)

3. **The dual ontology underlying NC:**
   - From the infinitisc scheme: math → phys law → information
   - To the finitistic scheme: information → math → phys law

4. **The Mutual Re-definition between Numbers and Processes (MRNP) and its applications in NC:**
   - Geometric Perceptron (AC) vs. Dynamic Perceptron (NC)
   - Extrinsic non-computable (AC) vs. Intrinsic computable (NC) chaotic dynamics characterization
   - Applications to cognitive neurosciences