Quantum Theory, namely the theory where Information and Physics meet

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Abstract: After more than a century since its birth, Quantum Theory still eludes our understanding. If asked to describe it, we have to resort to abstract and \textit{ad hoc} principles about complex Hilbert spaces. How is it possible that a fundamental physical theory cannot be described using the ordinary language of Physics? Here we offer a contribution to the problem, providing a short non-technical presentation of a recent derivation of Quantum Theory from information-theoretic principles [1]. The picture emerging from the principles is that Quantum Theory is the only theory of information compatible with the purity and reversibility of physical processes.

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1. Introduction

Quantum Theory is booming: It allows us to describe elementary particles and fundamental forces, to predict the colour of the light emitted by excited atoms and molecules, to explain the black body spectrum and the photoelectric effect, to determine the specific heat and the speed of sound in solids, to understand chemical and biochemical reactions, to construct lasers, transistors, and computers. This extraordinary experimental and technological success, however, is dimmed by huge conceptual difficulties. After more than hundred years from the birth of Quantum Theory, we still struggle to understand its puzzles and hotly debate on its interpretations. And even leaving aside the vexed issue of interpretations, there is a
more basic (and embarrassing) problem: We cannot even tell what Quantum Theory is without resorting
to the abstract language of Hilbert spaces! Compare quantum mechanics with the classical mechanics of
Newton and Laplace: Intuitive notions, such as position and velocity of a particle, are now replaced by
abstract ones, such as unit vector in a complex Hilbert space. Physical systems are now represented by
Hilbert spaces, pure states by unit vectors, and physical quantities by self-adjoint operators. What does
this mean? Why should Nature be described by this very special piece of mathematics?

It is hard not to suspect that, despite all our experimental and technological advancement, we are
completely missing the big picture. The situation was vividly portrayed by John Wheeler [2]: “Balancing
the glory of quantum achievements, we have the shame of not knowing “how come.” Why does the
quantum exist?”

The need for a more fundamental understanding was clear since the early days of Quantum Theory.
The first to be dissatisfied with the Hilbert space formulation was its founder himself, John von Neumann
[3]. Few years after the completion of his monumental book [4], von Neumann tried to understand
Quantum Theory as a new form of logics. His seminal work in collaboration with Birkhoff [5] originated
the field of quantum logics, which however did not succeed in producing a clear-cut picture capable to
cross the borders of a small community of specialists. More recently, a fresh perspective on the origin
of the quantum came from Wheeler. In his programme It from Bit, Wheeler argued that information
should be the fundamental notion in our understanding of the whole of physics: “all things physical are
information-theoretic in origin” [6]. If we accept this premise, then nothing is more natural then looking
for an information-theoretic understanding of quantum physics.

The idea that Quantum Theory is, in its backbone, a new theory of information became very concrete
with the raise of Quantum Information. This revolutionary discipline revealed that Quantum Theory is
not just a theory of unavoidable indeterminacy, as emphasized by its founders, but also a theory of new
exciting ways to process information, ways that were unimaginable in the old classical world of Newton
and Laplace. Quantum Information unearthed a huge number of operational consequences of Quantum
Theory: quantum states cannot be copied [7,8] but they can be teleported [9], the quantum laws allow for
secure key distribution [10,11], for fast database search [12], and for the factorization of large numbers
in polynomial time [13]. These facts are so impressive that one may be tempted to promote some of
them to the role of fundamental principles, and to derive the obscure mathematics of Quantum Theory
from the latter. The idea that Quantum Information could be the key to the mystery of the quantum
was enthusiastically championed by Fuchs [14] and Brassard [15] and rapidly led to a feverish quest for
new information-theoretic principles, like information causality [16], and to attempts to derive quantum
theory from informational ideas, like those of Refs. [17–21].

Recently, a new derivation of Quantum Theory from purely information-theoretic principles has been
presented in Ref. [1] (see also [22] for a short introduction to the background). In this work, which
marks a first step towards the realization of Wheeler’s dream, Quantum Information is shown to main-
tain its promise for the understanding of fundamental physics: Quantum Theory is now captured by a
complete set of information-theoretic principles, which can be stated using only the elementary language
of systems, processes, and probabilities. With respect to other related works, the new derivation offers a
clear-cut picture that nails down in few simple words what is special about of Quantum Theory: Quan-
tum Theory is, in the first place, a theory of information, which shares some basic features with classical
information, but differs from it on a crucial point, the *purity and reversibility of physical processes*. In a standard set of theories of information, Quantum Theory appears to be the only theory where the limited knowledge about the processes that we observe is enough to reconstruct a picture of the physical world where processes are pure and reversible. The purpose of this paper is to give a short, non-technical account of the informational principles of Quantum Theory and of the worldview that emerging from it. Particular emphasis will be given to the connection of the principles with other fundamental areas of theoretical physics.

### 2. A complete set of information-theoretic principles for Quantum Theory

To portray Quantum Theory we set up a scene where an experimenter, Alice, has many devices in her laboratory and can connect them in series and in parallel to build up circuits (Fig. 1). Alice’s circuits can be described with a graphical language were boxes represent different devices and wires represent physical systems travelling from one device to the next, as in the picturalist framework by Coecke [23].

![Figure 1. Alice’s laboratory.](image)

Any device has an input and an output system, and possibly some outcomes that Alice can read out. Each outcome labels a different process transforming the input into the output: the device itself can be viewed as a *random process*. Some devices have no input: they are *preparations*, which initialize the system in some state. Other devices have no output: they are *measurements*, which absorb the system and produce an outcome with some probability.

The features of the probability distributions arising in Alice’s experiments depend on the particular physical theory describing her laboratory: At this basic level, the theory could be classical or quantum, or any other fictional theory that we may be able to invent. We now start restricting the circle of possible theories: first of all, we make sure that Alice’s laboratory is not in a fictional Wonderland, but in a standard world enjoying some elementary properties common to Classical and Quantum Theory. The first property is:

**Principle 1 (Causality)** *The probability of an outcome at a certain time does not depend on the choice of experiments performed at later times.*

Causality ensures that Alice’s future choices do not affect the outcomes of her present experiments (*no-signalling from the future*).

Let us set more requirements on the processes taking place in Alice’s laboratory. For every random process, there is also a *coarse-grained process* where some random outcomes are joined together, thus...
A fine-grained process is instead a process where no information has been neglected: in this case Alice has maximal knowledge about the process taking place in her laboratory. For example, in the roll of a die the fine-grained processes are “the roll yielded the number \( n \)”, with \( n = 1, 2, 3, 4, 5, 6 \), while “the roll yielded an even number” is a coarse-grained process: When Alice declares outcome “even” she is joining together the outcomes 2, 4, and 6, thus neglecting the corresponding information. Our second principle is:

**Principle 2 (Fine-grained Composition)** The sequence of two fine-grained processes is a fine-grained process.

This principle establishes that “maximal knowledge of the episodes implies maximal knowledge of the history”: if Alice possesses maximal knowledge about all processes in a sequence, then she also possesses maximal information about the whole sequence. A physical theory where this did not hold would be highly pathological, because there would be some global information that cannot be accessed on a step-by-step basis.

For preparations, coarse-grained states are called *mixed* and fine-grained states are called *pure*. A pure state is a fine-grained preparation: Alice has maximal knowledge about the system’s preparation. A mixed state is the coarse-graining of some random preparation: Alice is ignoring (or choosing to ignore) some information about the preparation. A mixed state \( \rho \) is compatible with the system being prepared in any of the pure states from which \( \rho \) results as a coarse-graining.

**Principle 3 (Perfect distinguishability)** If a state is not compatible with some preparation, then it is perfectly distinguishable from some other state.

In other words, “possessing definite information about the preparation implies the ability to experimentally falsify some proposition”.

Suppose that Alice wants to transfer to another experimenter Bob the information she possesses about a system. If the system’s state \( \rho \) is mixed, then Alice ignores the exact preparation: with some probability the system could be in any of the pure states compatible with \( \rho \). Hence, the transmission should work for every pure state compatible with \( \rho \). Since transferring data has a cost, Alice would better *compress the information* (Fig. 2). The possibility of compression is stated by the following

![Figure 2. Compression.](image)

Our fourth principle guarantees the possibility of such a compression:
Principle 4 (Compression) *Information can be compressed in a lossless and maximally efficient fashion.*

Due to the compression principle, Alice can transfer information without transferring the particular physical system in which information is embodied.

The next principle concludes our list of requirements that are satisfied both by Classical and Quantum Theory:

Principle 5 (Local tomography) *The state of a composite system is determined by the statistics of local measurements on the components.*

Figure 3. Local Tomography. Alice can reconstruct the state of compound systems using only local measurements on the components. A world where this property did not hold would contain information that cannot be accessed with local experiments.

The five principles presented so far define a family of theories of information that can be regarded as standard. If it were just for these principles, Alice’s experiments could still be described, for example, by Classical Theory. What is then special about Quantum Theory? What makes it different from any other theory of information satisfying the five basic principles presented so far? Our answer is the following: Quantum Theory is the only theory of information that is compatible with a description of physical processes only in terms of pure states and reversible interactions. In a sense, Quantum Theory is the only physical theory of information: the only theory where Alice’s ignorance about processes happening in her laboratory is compatible with a complete picture of the physical world. Colourfully reinterpreting Einstein’s quote: God does not play dice, but we definitely do, and God must be able to describe our game!

Let us spell out our last principle precisely. In Quantum Theory, every random process can be simulated as a reversible interaction of the system with a pure environment (i.e. with an environment in a pure state). This simulation is essentially unique: once we fix the environment, two simulations of the same random process can only differ by a reversible transformation acting on the environment. Essential uniqueness is a very important feature: it means that Alice’s information about a random process happening in her laboratory is sufficient for her to determine the system-environment interaction in the most precise way possible (compatibly with the fact that Alice has no access to the environment). Distilling these ideas in a principle, we obtain the following:

Principle 6 (Purity and Reversibility of Physical Processes) Every random process can be simulated in an essentially unique way as a reversible interaction of the system with a pure environment.
This concludes our list. For finite systems (systems whose state is determined by a finite number of outcome probabilities) the six principles presented above describe Quantum Theory completely [1]: complex Hilbert spaces, superposition principle, Heisenberg’s uncertainty relations, entanglement, no-cloning, teleportation, violation of Bell’s inequalities, quantum cryptography—every quantum feature is already here, encapsulated in the principles. The detailed proof can be found in Ref. [1]. The surprising result here is that, although our sketch of Alice’s laboratory may seem too simplistic (after all, the Universe is not a big laboratory where we can choose the preparations and measurements at will!), this scenario is rich enough to capture Quantum Theory.

2.1. Conservation of Information and the Purification Principle

We now illustrate two important messages of the Purity and Reversibility Principle. The first message is that irreversibility can be always modelled as loss of control over an environment. In other words, the principle states a law of Conservation of Information according to which information can never be destroyed but can only be discarded. Here we are talking about information in a basic, non-quantitative sense: we mean information about the system’s preparation, which is encoded in the system’s state and is conserved whenever the system can be taken back to its initial state. If we regard the pieces of information carried by physical systems as fundamental blocks constituting our world, then the Conservation of Information is a must. Its importance, at least at the heuristic level, can be easily seen in the debate that followed Hawking’s discovery of the thermal radiation emitted by black holes [24]: The trouble with Hawking’s result was exactly that it seemed to negate the Conservation of Information [25]. In this case, the conviction that the Conservation of Information is fundamental led t’Hooft [26] and Susskind [27] to the formulation of the holographic principle, a major breakthrough in quantum gravity and string theory.

The second important message of the Purity and Reversibility Principle is that we can simulate every physical process using a pure environment, that is, without pumping entropy from the environment. Again, here we are talking about entropy in a very basic sense: whichever quantitative definition we may choose, entropy must be zero for pure states and non-zero for mixed states.

Purity and Reversibility can be expressed in an elegant way as Purification Principle: “every mixed state arises in an essentially unique way by discarding one component of a compound system in a pure state” [28]. In other words: the ignorance about a part is always compatible with the maximal knowledge about the whole. Our result can be then rephrased as: quantum theory is the unique theory of information where the ignorance about a part is compatible with the maximal knowledge about the whole. This result finally realizes the intuition expressed by Schrödinger with his prophetic words about entanglement: “I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.” [29] The compatibility of the ignorance about a part with the maximal knowledge about the whole is also the key idea in a recent proposal for the foundations of statistical mechanics [30].

3. Discussion and conclusions
Before concluding, some remarks are in order. First of all, it is important to stress that the principles in Ref. [1] are about the syntax of physical experiments, and not about their semantics. Questions like “What is an observer?” or “What is a measurement?” are not addressed by the principles: Ref. [1] does not aim to solve the measurement problem or any related interpretational issue. Furthermore, it is important to note that in our sketch of Alice’s laboratory there is no fundamental scale: no “far vs close”, nor “slow vs fast”. The Schrödinger equation (with some suitable Hamiltonian) is a consequence of the principles, but the actual value of the Plank’s constant $\hbar$ and the validity of the quantization rules are not. Finally, we emphasize that, although the derivation of Ref. [1] holds for finite systems, it is natural to expect that the principles therein identify Quantum Theory also in infinite dimension: in that case one has to take care of many technicalities, which however have more to do with the mathematical problem of infinity rather than with the conceptual problems of Quantum Theory.

In conclusion, building on the results of Ref. [1], in this paper we presented six informational principles that completely capture the world of Quantum Theory, which, for the first time since its birth, can now be described with the elementary language of Physics, without appealing to external ad hoc notions. The view emerging from the principles is that Quantum Theory is the only physical theory of information: the only theory where the limited information possessed by the experimenter is enough to construct a picture of the world where all states are pure and all processes are reversible.

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