Intelligence and reference. 
Formal ontology of the natural computation

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Abstract. In a seminal work published in 1952, “The chemical basis of morphogenesis” — considered as the true start point of the modern theoretical biology —, A. M. Turing established the core of what today we call “natural computation” in biological systems, intended as self-organizing dynamic systems. In this contribution we show that the “intentionality”, i.e., the “relation-to-object” characterizing biological morphogenesis and cognitive intelligence, as far as it is formalized in the appropriate ontological interpretation of the modal calculus (formal ontology), can suggest a solution of the reference problem that formal semantics is in principle unable to offer, because of Gödel and Tarski theorems. Such a solution, that is halfway between the “descriptive” (Frege) and the “causal” (Kripke) theory of reference, can be implemented only in a particular class of self-organizing dynamic systems, i.e., the dissipative chaotic systems characterizing the “semantic information processing” in biological and neural systems.

1 INTRODUCTION

1.2 Natural computation and algorithmic computation

Today natural computation (NC) is considered as an alternative paradigm to the algorithmic computation (AC) paradigm in natural and computer sciences, being the paternity of only the latter one generally ascribed to Alan Mathison Turing (1912-1954) seminal work. On the contrary, after the publication of his famous seminal work on algorithmic computation in 1936 [1] by the notions of Turing Machine (TM) and Universal Turing Machine (UTM), Turing worked for widening the notion of “computation” in the direction of what today we define as “natural computation”.

Before all, he defined the notion of Oracle-machine(s) – i.e., a TM enriched with the output of operations not computable by a TM, endowing the TM with the primitives of its computable functions – and of their transfinite hierarchy, in his doctoral work at Princeton, under the Alonso Church supervision, published in 1939 [2]. Afterward, in 1947 in a lecture given at the London Mathematical Society [3], and hence in an unpublished communication for the National Physical Laboratory in 1948 [4], he sketched the idea of computational architectures made by undefined interacting elements, that can be suitably trained, so to anticipate the so-called Artificial Neural Networks (ANN) computational architectures.

Finally, in a much more known contribution on a new mathematical theory of morphogenesis, published in 1952 [5], Turing was the first who studied a model of pattern formation via non-linear equations, in the specific case of chemical reaction-diffusion equations simulated by a computer.

This pioneering work on non-linear systems, and their simulation via computers, is, indeed, among all the pioneering works of Turing, the most strictly related with the new paradigm of NC, be-
1.3 Relevance of the reference problem in NC

In this paper, we want to suggest how a foundational approach to NC, overall as to its logical and semantic components cannot disregard the essential point of how to integrate in one only formalism the physical (“natural”) realm with the logical-mathematical (“computation”) one, as well as their relationship. That is, the passage from the realm of the causal necessity (“natural”) of the physical processes, to the realm of the logical necessity (“computational”), eventually taking them either in a sub-symbolic, or in a symbolic form. This foundational task can be performed, by the newborn discipline of theoretical formal ontology [8, 9, 10, 11, 12], as distinguished from formal ontology engineering – an applicative discipline, well established and diffused in the realm of computational linguistics and semantic databases.

Particularly, the distinction between formal logic and formal ontology is precious for defining and solving foundational misunderstanding about the notion of reference that the NC approach had the merit of emphasizing, making aware of it the largest part of the computer science community – and also the rest, we hope, of the scientific community, as far as NC is spreading all over the entire realm of the natural sciences.

In fact, as A. Tarski rightly emphasized since his pioneering work on formal semantics [13], not only the meaning but also the reference in logic has nothing to do with the real, physical world. To use the classic Tarski’s example, the semantic reference of the true statement “the snow is white” is not the whiteness of the crystalized water, but at last an empirical set of data to which the state-ment is referring, eventually taken as a sub-symbolic, or in a symbolic form. This foundational task can be performed, by the newborn discipline of theoretical formal ontology [8, 9, 10, 11, 12], as distinguished from formal ontology engineering – an applicative discipline, well established and diffused in the realm of computational linguistics and semantic databases.

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In this sense, Putnam stated, we would have to consider ultimately names as rigid designators “one - to - one” of objects in S. Kripke’s sense [21]. But no room exists, also in Kripke’s theory of partial reference – and hence using Kleene’s genius solution of partial recursive predicates for dealing with the problem of enumeration of partial functions, that Gödelian notion of general recursion cannot approach in principle (see [22, p. 313 and 327f.]) – for defining the notion of rigid designation in terms of a purely logical relation, since any logical relation only holds among tokens and not between tokens and objects, as Tarski reminded us. Hence a formal language has always to suppose the existence of names (or numbers) as rigid designators and cannot give them a foundation.

To explain by an example the destructive consequences of this point for a functionalist theory of mind, Putnam suggested a sort of third version of the famous “room – metaphor” [23, p. 116ff.], after the original “Turing test” version of this metaphor, and J. Searle’s “Chinese – room” version of it. Effectively, Putnam proposed by his metaphor a further test that a TM cannot solve and that has much deeper implications than the counterexample to the Turing test proposed by Searle. For instance, Putnam said, if we ask “how many objects are in this room?”, the answer supposes a previous decision about which are to be considered the “real” objects to be enumerated — i.e., rigidly designated by numerical units. So, one could answer that the objects in that room are only three (a desk, a chair and a lamp over the desk). However, by changing the enumeration axiom, another one could answer that, for instance, the objects are many billions, because we have to consider also the molecules of which the former objects are constituted.

Out of metaphor, any computational procedure of a TM (and any AC procedure at all, if we accept Church’s thesis) supposes the determination of the basic symbols on which the computations have to be carried on – the partial domain on which the recursive computation has to be carried on. Hence, from the semantic standpoint, any computational procedure supposes that such numbers are encoding (i.e., unambiguously naming as rigid designators) as many “real objects” of the computation domain. In short, owing to the coding problem, the determination of the basic symbols (numbers) on which the computation is carried on, cannot have any computational solution in the AC paradigm.

The closed, stand-alone character of AC models depends thus on the purely syntactic and semantic level in which the logical approach can develop its analysis of the reference problem, hence at a necessarily representational/symbolic level. Precisely for this systematic impossibility of the logical theory of reference of justifying logical truth as adequacy to outer reality H. Putnam abandoned the functionalist approach to cognitive science he himself contributed to define in 60’s of last century, for an intentional, non-representational theory of a cognitive act, based on a causal theory of reference as anticipated in his early works of 1973 [24] and of 1975 [15], even though in a different sense as to other representatives of this theory like, for instance, K. S. Donnellan [25] and S. Kripke himself [21]. Putnam indeed rightly vindicated that a causal theory of reference supposes that at least at the beginning of the social chain of “tradition” of a given denotation there must be an

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4 A typical representative of researcher in both fields of formal ontology is Barry Smith (see, for instance, [40, 41, 42]).
effective causal relation from the denoted thing to (the cognitive agent producing) the denoting name – and, in the limit, in this causal sense must be intended also the act of perception Kripke vindicated as sufficient for the dubbing of a given object. To synthesize this position, even though Putnam never spoke in these terms, what is necessary is a “causal”, “finiteistic” theory of coding in which the real thing causally and progressively determines the partial domain of the descriptive function recursively denoting it. It is thus evident the necessity of formal ontology for formalizing such an approach to the meaning/reference problem in the NC paradigm. That is, it is evident the necessity of a formal calculus of relations able to include in the same, coherent, formal framework both “causal” and “logical” relations, as well as the “pragmatic” (real, causal relations with the cognition/communication agents), and not only the “syntactic” (logical relations among terms) “semantic” (logical relations among symbols) components of meaningful actions/computations/cognitions.

2 FROM FORMAL LOGIC TO FORMAL ONTOLOGY

2.1 Extensional vs. intensional logic

The modal logic with all its intensional interpretations are what is today defined as philosophical logic [26], as far as it is distinguished from the mathematical logic, the logic based on the extensional calculus, and the extensional meaning, truth, and identity3. What is new is that also the intensional logics can be formalized (i.e., translated into a proper symbolic language, and axiomatized), against some rooted prejudices among “continental” philosophers, who abhor the symbolic hieroglyphics of the “analytic” ones. I.e., there exists an intensional logical calculus, just like there exists an extensional one, and this explains why both mathematical and philosophical logic are today often quoted together within the realm of computer science. This means that classical semantic and even the intentional tasks can be simulated artificially. This is the basis of the incoming “Web3 revolution”, i.e., the advent of the semantic web. Hence, the “thought experiment” of Searle’s “Chinese Room” is becoming a reality, as it happens often in the history of science.

Anyway, to conclude this part, the main intensional logics with which we are concerned in the present paper are:

1. Alethic logics: they are the descriptive logics of “being/not being” in which the modal operators have the basic meaning of “necessity/possibility” in two main senses:
   a. Logical necessity: the necessity of lawfulness, like in deductive reasoning

2. The deontic logics: concerned with what “should be or not should be”, where the modal operators have the basic meaning of “obligation/permission” in two main senses: moral and legal obligations.

3. The epistemic logic: concerned with what is “science or opinion”, where the modal operators have the basic meaning of “certainty/uncertainty”. It is evident that all the “belief” logic pertains to the epistemic logic, as we see below.

2.2. Interpretations of modal logic

For our aims, it is sufficient here to recall that formal modal calculus is an extension of classical propositional, predicate and hence relation calculus with the inclusion of some further axioms. Here, we want to recall only some of them — the axioms N, D, T, 4 and 5 —, useful for us: N: \( \langle X \rightarrow \alpha \rangle \Rightarrow \langle \Box X \rightarrow \Box \alpha \rangle \), where \( X \) is a set of formulas (language), \( \Box \) is the necessity operator, and \( \alpha \) is a meta-variable of the propositional calculus, standing for whichever propositional variable \( p \) of the object-language. N is the fundamental necessitation rule supposed in any normal modal calculus

D: \( \langle \Box \alpha \rightarrow \alpha \rangle \), where \( \Box \) is the possibility operator defined as \( \neg \Box \neg \alpha \). D is typical, for instance, of the deontic logics, where nobody can be obliged to what is impossible to do.

T: \( \langle \alpha \rightarrow \alpha \rangle \). This is typical, for instance, of all the alethic logics, to express either the logic necessity (determination by law) or the ontic necessity (determination by cause).

4: \( \langle \Box \alpha \rightarrow \Box \alpha \rangle \). This is typical, for instance, of all the “unification theories” in science where any “emergent law” supposes, as necessary condition, an even more fundamental law.

5: \( \langle \alpha \rightarrow \Box \alpha \rangle \). This is typical, for instance, of the logic of metaphysics, where it is the “nature” of the objects that determines necessarily what it can or cannot do.

By combining in a consistent way several modal axioms, it is possible to obtain several modal systems which constitute as many syntactical structures available for different intensional interpretations. So, given that \( K \) is the fundamental modal systems, given by the ordinary propositional calculus \( p \) plus the necessitation axiom \( N \), some interesting modal systems are for our aims are: KT4 (S4, in early Lewis’ notation), typical of the physical ontology; KT45 (S5, in early Lewis’ notation), typical of the metaphysical ontology; KD45 (Secondary S5), with application in deontic logic, but also in epistemic logic, in ontology, as \( p \) and hence in NC as we see.

Generally, in the alethic (either logical or ontological) interpretations of modal structures the necessity operator \( \Box p \) is interpreted as “\( p \) is true in all possible world”, while the possibility operator \( \Box p \) is interpreted as “\( p \) is true in some possible world”. In any case, the so called reflexivity principle for the necessity operator holds in terms of axiom T, i.e., \( \Box p \rightarrow p \). In fact, if \( p \) is true in all possible

3 What generally characterizes intensional logic(s) as to the extensional ones is that neither the extensionality axiom – reducing class identity to class equivalence, i.e., \( A \rightarrow B \Rightarrow A = B \) - nor the existential generalization axiom – \( P a \Rightarrow \exists x P x \), where \( P \) is a generic predicate, \( a \) is an individual constant, \( x \) is an individual variable – of the extensional predicate calculus hold in intensional logic(s). Consequently, also the Fregean notion of extensional truth based on the truth tables holds in the intensional predicate and propositional calculus. Of course, all the “first person” (both singular, in the case of individuals, and plural, in the case of groups), i.e., the belief or intentional (with \( t \) statements, belong to the intensional logic, as Searle, from within a solid tradition in analytic philosophy [45, 46, 47], rightly emphasized [39, 38]. For a formal, deep characterization of intensional logics as to the extensional ones, from one side, and as to intensionality, from the other side, see [48].
worlds, it is true also in the *actual* world (E.g., “if it is necessary that this heavy body falls (because of Galilei’s law), then this body really falls”).

This is not true in *deontic* contexts. In fact, “if it is obligatory that all the Italians pay taxes, does not follow that all Italians really pay taxes”; i.e., $Op \not\rightarrow p$, where $O$ is the necessity operator in deontic context. In fact, the obligation operator $Op$ must be interpreted as “$p$ is true in all *ideal worlds*” different from the actual one, otherwise $O \equiv \Box_i$, i.e., we are in the realm of metaphysical determinism where freedom is an illusion and ethics too. The reflexivity principle in deontic contexts, able to make obligations really effective in the actual world, must be thus interpreted in terms of an *optimality operator* $Op$ for intentional agents, i.e.,

$$
(Op \rightarrow p) \iff ((Op \rightarrow x,p) \land c_i \land c_n) \rightarrow p
$$

Where $x$ is an intentional agent, $c_i$ is an acceptance condition and $c_n$ is a non-impediment condition.

In similar terms, in *epistemic* contexts, where we are in the realm of representations of the real world. The interpretations of the two modal epistemic operators $B(x,p)$, “believes that $p'$, and $S(x,p)$, “$x$ knows that $p$” are the following: $B(x,p)$ is true iff $p$ is true in the realm of representations believed by $x$. $S(x,p)$ is true iff $p$ is true for all the *founded* representations believed by $x$. Hence the relation between the two operators is the following:

$$S(x,p) \iff (B(x,p) \land F)$$

(1)

Where $F$ is a *foundation relation*, outside the range of $B$, and hence outside the range of $x$ consciousness, otherwise we should not be dealing with “knowing” but only with a “believing of knowing”. I.e., we should be within the realm of solipsism and/or of metaphysical nihilism, systematically reducing “science” or “well founded knowledge” to “believing”. So, for instance, in the context of a *logician* ontology, such a $F$ is interpreted as a supposed actually infinite capability of human mind of attaining the logical truth [27]. We will offer, on the contrary, a different *finitistic* interpretation of $F$ within NC. Anyway, as to the reflexivity principle in epistemic context,

$$B(x,p) \not\rightarrow p$$

In fact, believing that a given representation of the actual world, expressed in the proposition $p$, is true, does not mean that it is *effectively* true, if it is not well *founded*. Of course, such a condition $F$ — that hence has to be an *onto-logical condition* — is by definition satisfied by the operator $S$, the operator of sound beliefs, so that the reflexivity principle for epistemic context is given by:

$$S(x,p) \rightarrow p$$

(2)

2.3 Kripke’s relational semantics

Kripke relational semantic is an evolution of Tarski formal semantics, with two specific characters: 1) it is related to an *intuitionistic logic* (i.e., it considers as non-equivalent excluded middle and contradiction principle, so to admit coherent theories violating the first one), and hence 2) it is compatible with the *necessarily incomplete* character of the formalized theories (i.e., with Gödel theorems outcome), and with the *evolutionary character* of natural laws not only in biology but also in cosmology. In other terms, while in Tarski classical formal semantics, the truth of formulas is concerned with the state of affairs of *one only actual world*, in Kripke relational semantics the truth of formulas depends on states of affairs of worlds different from the actual one (= possible worlds). On the other hand, in contemporary cosmology is nonsensical speaking of an “absolute truth of physical laws”, with respect to a world where the physical laws cannot be always the same, but have to evolve with their referents [28, 29].

Anyway, the notion of “possible world” in Kripke semantics has not only a physical sense. On the contrary, as he vindicated many times, the notion of “possible world”, as syntactic structure in a relational logic, has as many senses as the semantic models that can be consistently defined on it. In Kripke words, the notion of “possible world” in his semantics has a *purely stipulatory character*. In the same way, in Kripke semantics, like the notion of “possible world” can be interpreted in many ways, so also the relations among worlds can be given as interpretations of the only relation of *accessibility*. In this way, a unified theory of the different intensional interpretations (alethic, ontology included, deontic, epistemic, etc.) of modal logic became possible, as well as a graphic representation of their relational semantics.

The basic notion for such a graphic representation is the notion of *frame*. This is an ordered pair , $<W, R>$, constituted by a domain $W$ of possible worlds $\{u, v, w, . . .\}$, and by a two-place relation $R$ defined on $W$, i.e., by a set of ordered pairs of elements of $W$ $(R \subseteq W \times W)$, where $W \times W$ is the *Cartesian product* of $W$ per $W$. E.g. with $W = \{u, v, w\}$ and $R = \{uRv\}$, we have:

![Diagram](image-url)

(3)

According to such a model, the accessibility relation $R$ is only in the sense that $v$ is accessible by $u$, while $w$ is not related with whichever world. If in $W$ all the worlds were reciprocally accessible, i.e., $R^e = \{uRv, vRw, uRw, wRu, wRv, vRw\}$, then we would have $R$ only included in $W \times W$. On the contrary, for having $R^e=W \times W$, we need that each world must be related also with itself, i.e.:

![Diagram](image-url)

(4)

Hence, from the standpoint of the relation logic, i.e., by interpreting $\{u, v, w\}$ as elements of a class we can say that this frame represents an *equivalence class*. In fact, a $R$, *transitive*, *symmetric* and *reflexive* relation holds among them. Hence, if we consider also the *serial relation*: $<(om u)(ex v)(uRv)>^6$, where “om” and “ex” are the meta-linguistic symbols, respectively of the universal and existential quantifier, we can discuss also the particular *Euclidean relation* that can be described in a Kripke frame. The Euclidean property generally in mathematics means a weaker form of the transitive property (that is, if one element of a set has the same relation with other two, these two have the same relation with each other).

I.e., $<(om u)(om v)(om w)(uRv \land uRw \Rightarrow vRw)>^6$:

![Diagram](image-url)

(5)

Where $et$ is the meta-symbol for the logical product.

Hence, for seriality, it is true also $<(om u)(om v)(uRv \Rightarrow vRv)>^6$:

![Diagram](image-url)

(6)

For ontological applications it is to be remembered that seriality means in ontology that the causal chain is always closed, as it is requested in physics by the first principle of thermodynamics, and in metaphysics by the notion of a first cause of everything.
Moreover, \(<\text{om } u\text{)} (\text{om } v\text{)} (\text{om } w) (uRv \text{ et } uRv \Rightarrow vRw \text{ et } wRv)>
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\text{a finitistic computational procedure always convergent in polynomial time, even in chaotic dynamics [30].}
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2.4 Double saturation S/P and its implementation

What characterizes the definite descriptions in a naturalistic formal ontology since of its Middle Age ancestors is the theory of double saturation between Subject and Predicate (S/P), driven by a causal action from the referential object. So Thomas Aquinas (1225-1274)\(^7\) depicts his causal theory of reference:

Science, indeed, depends on what is object of science, but the opposite is not true: hence the relation through which science refers to what is known is a causal [real not logical] relation, but the relation through which what is known refers to science is only logical [rational not causal]. Namely, what is knowable (scibile) can be said as “related”, according to the Philosopher, not because it is referring, but because something else is referring to it. And that holds in all the other things relating each other like the measure and the measured, … (Q. de Ver., 21. 1. Square parentheses and italics are mine).

In another passage, this time from his commentary to Aristotle book of Second Analytics, Aquinas explains the singular reference in terms of a “one-to-one universal”, as opposed to “one-to-many universals” of generic predications.

It is to be known that here “universal” is not intended as something predicated of many subjects, but according to some adaptation or adequation (adaptationem vel adequationem) of the predicate to the subject, as to which neither the predicate can be said without the subject, nor the subject without the predicate (In Post.Anal., 1,xi,91. Italics mine).

So, Aquinas’ idea is that the predicative statement, when applied denoting a singular object must be characterized by a “mutual re-definition” between the subject \(S\) and the predicate \(P\) “causally” driven by the referential object itself. A procedure formalized in Kripke’s frame (4) and that A. L. Perrone first demonstrated being

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\[\text{[7]}\text{ G. Dodig-Crnkovic, “Physical computation as dynamics of form that glues everything together” Information, vol. 3, pp. 204-218, 2012.}
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\[\text{[8]}\text{ N. B. Cocchiarella, "Logic and ontology," Axiomathes, vol. 12, pp. 117-150, 2001.}
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\[\text{[9]}\text{ N. B. Cocchiarella, Formal Ontology and Conceptual Realism, Berlin-New York: Springer Verlag, 2007.}
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\[\text{[10]}\text{ G. Basti, “Ontologia formale: per una metafisica post-}
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\text{Historically, he first introduced the notion and the term of “intention” (intento) in the epistemological discussion, in the context of his naturalistic ontology. The approach was hence rediscovered in the XIX century by the philosopher Franz Brentano, in the context of a conceptualist ontology, and hence passed to the phenomenological school, throug Brentano’s most famous disciple: Edmund Husserl.}
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Finally, if we see at the last two steps, we are able to justify, via the Euclidean relation, a set of secondary reflexive and symmetrical relations, so that we have the final frame of a secondary equivalence relation among worlds based on an Euclidean relation with a third one:

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Of course, this procedure of equivalence constitution by a transitive and serial (=causal) relation can be iterated indefinitely:

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3 CONCLUSIONS

To conclude, it is important to emphasize that the frame (3) and the frame (4) are a graphic representation in Kripke’s approach of S5 and KD45 modal systems respectively. If we see at these frames, we can understand immediately, why, from one side; S5 is the only axiomatic system in modal logic, since all its elements constitutes one only equivalence class. On the other side, we can also understand immediately also why KD45 is named “secondary S5”. In fact, \(<v,w>\text{ in (4) and }<v,w,z>\text{ in (5) constitute two equivalence classes via their Euclidean relation with }<w>\text{.}

If S5 is thus, in ontology, the common syntactic structure of all possible metaphysics, KD45 is the common structure of any ontology of the emergence of a new natural law (and hence of a new natural kind) from more fundamental levels of physical causality. Moreover, in epistemic logic, if \(<u>\text{ represents the referential object, then }<v,w>\text{ and/or }<v,w,z>\text{ represent two equivalence classes of two and/or three symbols co-denoting it. Two classes of logical symbols with their relations constituted via a common causal relation from the denoted object! It is evident that such a solution of the reference problem can be implemented only in a NC model where }<v,w,z>\text{ can be interpreted as cognitive agents, particularly as (pseudo-)cycles of the same chaotic attractor, as we and others demonstrated elsewhere (See [30, 18, 31, 32]; [33, 34, 35]).}

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